CE142 Assignment: The Matrix Exponential

Matlab Code for the Matrix Exponential Function

This Matlab code is designed to find the matrix exponential through iteration of the power series; it takes two arguments, the matrix, and N, which is the number of iterations of the power series that the function is to compute (The more iterations, the closer to the true value it will be). The function then compares the number of rows and number of columns and returns 'Matrix must be square' if they are not equal. 'B' is then initialised as an identity matrix of the same size as A and factn is initialised as 1 (which is 1!). A for loop iterates N times, calculating the next required factorial, then adding (A^n)/factn (the subsequent term in the power series) to the total of B.

When:

J=[0,-1;1,0] K=[0,0;1,1] and L=[2,1;-1,2]

matexp(J,25)= [.5403,-.8415;.8415,.5403] matexp(K,25)= [1,0;1.7183,2.7183]

matexp(L,25)= [3.9923,6.2177;-6.2177,3.9923] matexp(J+K,25)= [-.1995,-1.2082,2.4164,1.0087]

**matexp(J+L,25)= [7.3891,0;0,7.3891]** matexp(K+L,25)= [7.3891,12.6965;0,20.0855]

The reason matexp(J,25) does not equal [0,e^-1;e,0] is because you are not finding the exponential of the individual values (a property of diagonal matrices, whereas this is off-diagonal) in the matrix but rather a series which includes A^n, which will change the values in the matrix with every term.

JK=[0,-1;1,0][0,0;1,1]=[-1,-1;0,0] KJ=[0,0;1,1][0,-1;1,0]=[0,0;1,-1]

**JL=[0,-1;1,0][2,1;-1,2]=[1,-2;2,1] LJ=[2,1;-1,2][0,-1;1,0]=[1,-2;2,1]**

expJexpK=[.5403,-.8415;.8415,.5403][1,0;1.7183,2.7183]=[-.9056,-2.2874;1.7698,1.4686]

expKexpJ=[1,0;1.7183,2.7183][.5403,-.8415;.8415,.5403]=[.5403,-.8415;3.2158,.0227]

**expJexpL=[.5403,-.8415;.8415,.5403][3.9923,6.2177;-6.2177,3.9923]=[7.3892,0;0,7.3892]**

**expLexpJ=[3.9923,6.2177;-6.2177,3.9923][.5403,-.8415;.8415,.5403]=[7.3892,0;0,7.3892]**

The examples highlighted above (using the matrices J and L) satisfy the equation:

exp(A+B)=expAexpB=expBexpA

This is a property of matrices that are commutable ie AB=BA

When:

F=[1,0,0;0,2,0;0,0,3] G=[1,0,0;0,0,1;0,1,0] H=[1,-1,0;1,0,1;0,-1,1]

expF= [2.7183,0,0;0,7.3891,0;0,0,20.855] expG=[2.7183,0,0;0,1.5431,1.1752;0,1.1752,1.5431]

expH=[1.8635,-1.2082,-.8548;1.2082,-.1995,1.2082;-.8548,-1.2082,1.8635]

Tuned Circuits

This section shows how you can analyse circuits and represent equations as matrices.

d/dt [V;i]=[0,2;-1,-R][V;i]=[2i;-V-Ri]

**A B**

[V(t);i(t)]=exp(t[0,2;-1,-R])[V[0];i(0)]

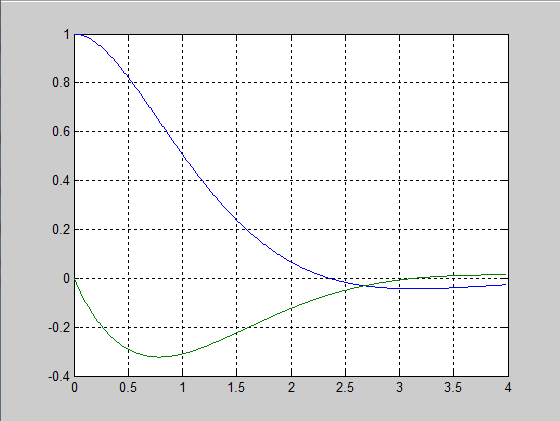
As this is a function of a function, d/dt =A'B+B'A

A'=(e^tA)A B'=0 Therefore d/dt =(e^tA)AB When t=0 this is simply AB

The code in this section firstly initialises A as the 2x2 matrix stated above (with 3 as the value of R). N=400 is used to make t a 1x400 matrix of zeroes and vi as a 2x400 matrix; the first column of vi is then changed to [1;0], with the ':' in that statement means from the beginning. A for loop then iterates N times, firslty making t increment in hundredths from 0 to 4 seconds. It then calculates (e^tA)\*[V(0);i(0)] and stores the value in [V(t);i(t)]

The blue line is voltage, green is current. In this graph, voltage will always be positive and current will always be negative. This is because in the voltage equation 2e^(-t)-e^(-2t), -e^(-2t) is always the larger of the two values, and for current, e^(-2t)-e^(-t), the first value is always smaller, so it could never be positive.

To modify the code to so the resistor is 2 ohms, simply replace the last value in A with -2.

As both voltage and current are an exponential multiplied by a sine value, the signs of voltage and current will change as sine goes between 1 and -1

Diagonalisation

When C=[1,0;0,3] and D=[-1,0;0,-2]

CD=[-1,0;0,-6] DC=[-1,0;0,-6] C^2=[1,0;0,9] D^2=[1,0;0,4]

When multiplying diagonalised matrices, you can use conventional multiplication with the corresponding elements to get the correct result. The exponential matrix is also [e^C1,0;0,e^C4]

U=[2,1;-1,-1] U^-1=[1,1;-1,2]

D=U^-1AU =[1,1;-1,-2]([0,2;-1,-3][2,1;-1,-1])=[1,1;-1,-1][-2,-2;1,2]=[-1.0;0,-2]=D

A=UDU^-1 =[2,1;-1,-1]([-1,0;0,-2][1,1;-1,-1])=[2,1;-1,-1][-1,-1;2,4]=[0,2;1,-3]=A

Exp(tA)=Uexp(tD)U^-1

tD=t[-1.0;0,-2=[-t,0;0,-2t] Due to its diagonal nature, this means that exp(tD)=[exp(-t),0;0,exp(-2t)]

d/dt =(e^tA)AB from the last section can then be changed to:

[V(t);i(t)]=U[exp(-t),0;0,exp(-2t)]U^-1[1;0] with [1;0] being he value [V(0);i(0)]

Multiplying out these matrices:

U^-1[1;0]=[1,1;-1,-2][1;0]=[1;-1]

[e^-t,0;0,e^-2t][1;-1]=[e^-t;-e^-2t]

[2,1;-1,-1][e^-t;-e^-2t]=[2e^(-t)-e^(-2t);e^(-2t)-e^(-t)]=[V(t);i(t)]

These equations match the values on the graph plotted earlier:

2e^(-1.5)-e^(-3)=0.3964

e^(-3)-e(-1.5)=-0.1733

When V=[1+j,1-j;-1,-1] V^-1=[-.5j,-.5-.5j;.5j,-.5+.5j]

E=V^-1BV=[-.5j,-.5-.5j;.5j,-.5+.5j][0,2;-1,-2] [1+j,1-j;-1,-1]= [.5+.5j,1;.5j-.5j,1] [1+j,1-j;-1,-1]

=[-1+j,0;0,-1-j]

*Working:*

*Ea=(.5+.5j)( 1+j)+-1=(.5-.5+.5j+.5j)+-1=-1+j*

*Eb=(.5+.5j)( 1-j)+-1=(.5-.5j+.5j-.5j^2)+-1=0*

*Ec=(.5-.5j)( 1+j)+-1=(.5+.5j-.5j-.5j^2)+-1=0*

*Ed=(.5-.5j)( 1-j)+-1=(.5-.5j-.5j+.5j^2)+-1=-1-j*

B=VEV^-1=[0,2;-1,-2] [-1+.5j,0;0,-1-j] [-.5j,-.5-.5j;.5j,-.5+.5j]

=[-2,-2;1-j,1+j] ] [-.5j,-.5-.5j;.5j,-.5+.5j]=[0,2;-1,-2]=B

*Working:*

*VEa=(1+j)(- 1+j)=-1+j-j+j^2=-2*

*VEb=(1-j)( -1-j)=-1-j+j+j^2=-2*

*Ba=(-2\*-.5j)+(-2\*.5j)=0*

*Bb=(-2\*(-.5-.5j))+(-2\*(-.5+.5j))=2*

*Bc=(1-j)(-.5j)+(1+j)(.5j)=(-.5j-.5)+(.5j-.5)=-1*

*Bd=(1-j)(-.5-.5j)+(1+j)(-.5+.5j)=(-.5-.5j+.5j-.5)+(-.5+.5j-.5j-.5)=-2*

[V(t);i(t)]=Exp(-t)V[cos(t)+jsin(t),0;0,cos(t)-jsin(t)]V^-1[1;0]

[-.5j,-.5-.5j;.5j,-.5+.5j][1;0]=[-.5j;.5j]

[cos(t)+jsin(t),0;0,cos(t)-jsin(t)] [-.5j;.5j]=[-.5jcos(t)+.5sin(t);.5jcos(t)+.5sin(t)]

[1+j,1-j;-1,-1] [-.5jcos(t)+.5sin(t);.5jcos(t)+.5sin(t)]=[sin(t);-sin(t)]

Exp(-t)[sin(t);-sin(t)]=[e^(-t)sin(t);-e^(-t)sin(t)]

This agrees with second graph I obtained:

e^-1.5\*sin(1.5)=0.22

-e^-1.5\*sin(1.5)=-0.22

*Working:*

*(1+j)(-.5jcos(t)+.5sin(t))+(1-j)(.5jcos(t)+.5sin(t))*

*=(.5cos(t)-.5jcos(t)+.5sin(t)+.5jsin(t))+(-.5cos(t)+.5jcos(t)+.5sin(t)-.5jsin(t))=sin(t)*

*(-1)(-.5jcos(t)+.5sin(t))+(-1)(.5jcos(t)+.5sin(t))=.5jcos(t)-.5sin(t)-.5jcost(t)-.5sin(t)=-sin(t)*